TELFORD JUNIOR SCHOOL

Calculation Policy,

LOWER KS2

FEBRUARY 2022

CONCRETE, PICTORAL, ABSTRACT

Children (and adults!) can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems.

Concrete step of CPA

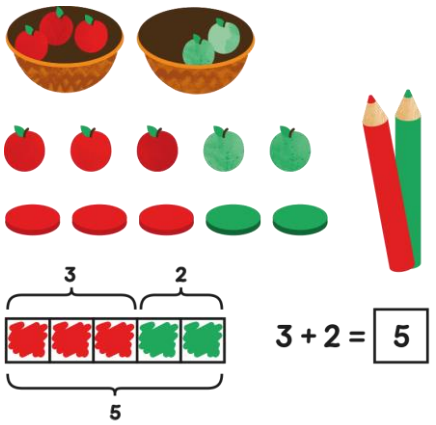
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.



Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, −, x, /) to indicate addition, multiplication or division.

A skilled teacher will go back and forth between each stage to reinforce concepts.

Teachers should **vary** the apparatus that children use in class. For example, students might one day use counters, another day they might use a ten frame. Children should be encouraged to represent maths problems in a variety of ways. For example, drawing an array, a number bond diagram or a bar model.

By systematically varying the apparatus and methods used to solve a problem, children can craft powerful mental connections between the concrete, pictorial, and abstract phases.

When teaching young children numbers, counters and multi-link cubes are more commonly used. However, concrete materials are frequently shelved by the time children reach KS2 — many teachers believe them to be too childish or distracting. Removing concrete materials exposes children to abstract concepts too early. As a result, they miss out on the opportunity to build a conceptual mathematical understanding that can propel them through their education.

It is important to recognise that the CPA model is a progression. By the end of KS1, children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

NB: Our CPA policy follows on exactly from the one used at Telford Infant School.

All Calculation Policies are on the Maths padlet:

https://padlet.com/powells16/7drkb06hinnwkslo

NUMBER AND CALCULATION: UNDERSTANDING KEY MATHS TERMS

CARDINALITY

The cardinal value of a number refers to the quantity of things it represents, e.g. the numerosity, 'howmanyness', or 'threeness' of three. When children understand the cardinality of numbers, they know what the numbers mean in terms of knowing how many things they refer to. Counting is one way of establishing how many things are in a group, because the last number you say tells you how many there are. Children enjoy learning the sequence of counting numbers long before they understand the cardinal values of the numbers. Subitising is another way of recognising how many there are, without counting.

Typical progression of key ideas in this concept

Counting: saying number words in sequence

Counting: tagging each object with one number word

Counting: knowing the last number counted gives the total so far

Subitising: recognising small quantities without needing to count them all

Numeral meanings

Conservation: knowing that the number does not change if things are rearranged (so long as none have been added or taken away)

SUBITISING

Subitising is recognising how many things are in a group without having to count them one by one. Children need opportunities to see regular arrangements of small quantities, e.g. a dice face, structured manipulatives, etc., and be encouraged to say the quantity represented. Children also need opportunities to recognise small amounts (up to five) when they are not in the 'regular' arrangement, e.g. small handfuls of objects.

UNITISING

How can you quickly work out the number of eggs contained in a stack of egg boxes? How would you calculate the total amount of money in a pile of 50p pieces? You'd probably count the boxes, or the coins, and then do a multiplication.

The mathematical term for counting the egg boxes (rather than opening the boxes and counting individual eggs) is 'unitising'. This means treating groups that contain, or represent, the same numbers of things as 'units' or 'ones'.

Being able to 'unitise' is fundamental in handling money and in understanding place value. It forms a thread of understanding throughout further development of multiplication and division concepts. It allows children to move from additive to multiplicative thinking.

COMMUTATIVITY

Commutativity is used in Maths equations and describes sums that can be moved around and will still give the same answer. 'Commutative' comes from the word 'commute' which means to move and travel around, so equations that are commutative have numbers that can be moved within the equation.

Addition and multiplication are commutative. When two numbers are added or multiplied, this can be done in any order and the same answer will be obtained.

Addition example: 3 + 2 = 5, 2 + 3 = 5 Multiplication example: 4 x 6 = 24, 6 x 4 = 24.

Subtraction and division are not commutative as the answers will be different if numbers are swapped around.

PARTITIONING

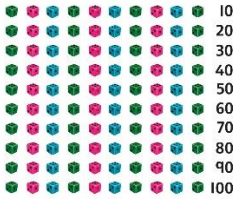
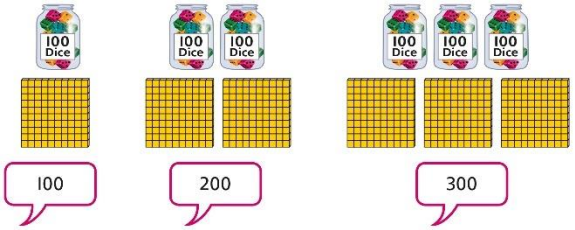
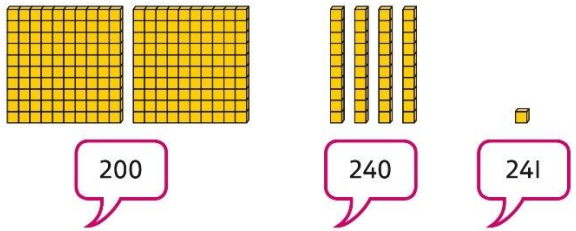
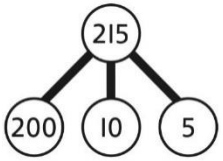
Partitioning a number is splitting a number into parts to aid calculations.

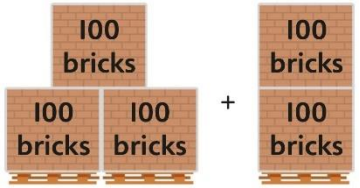
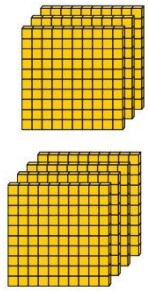
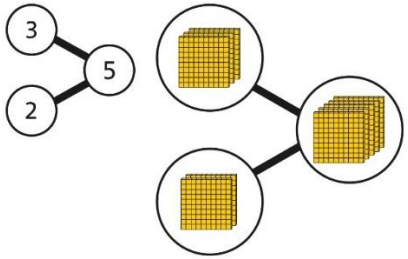
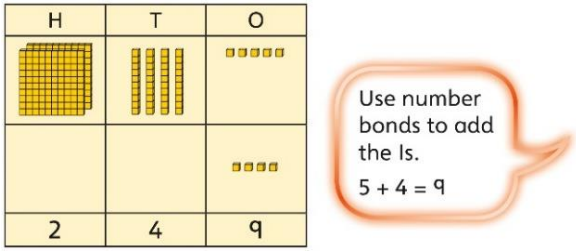
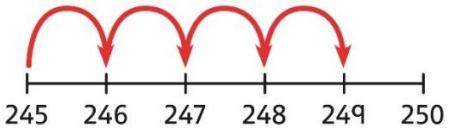
Sometimes it will be **splitting** the number according to **place value** E.g. 7645 = 7000 + 600 + 40 + 5

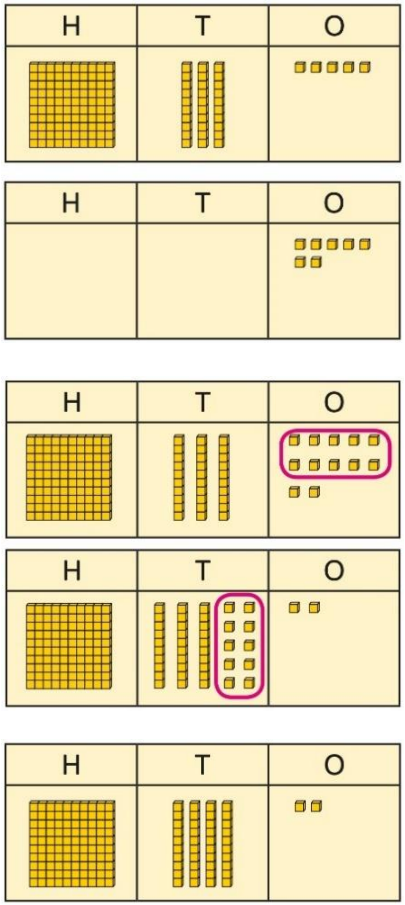
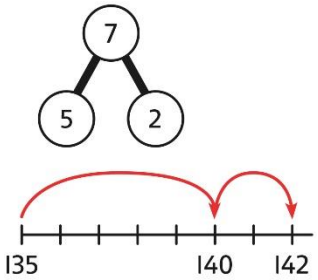
At other times numbers will be **split** in order to use **known facts**

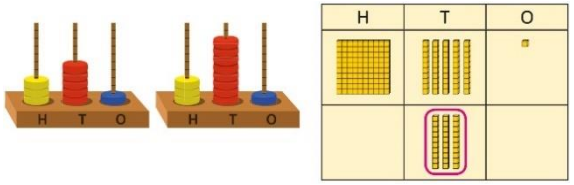
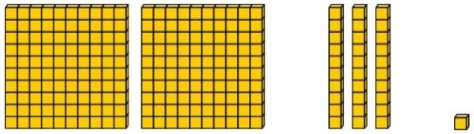
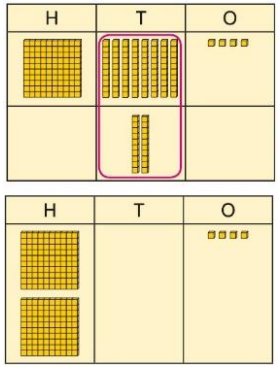
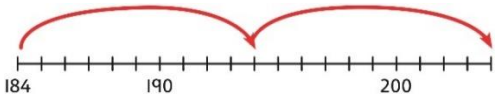
E.g. to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.

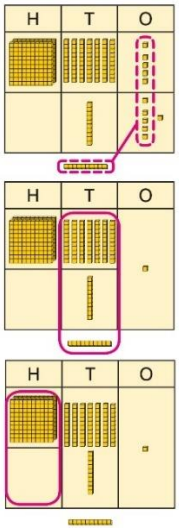
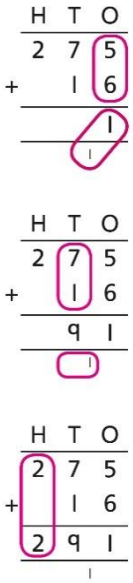
KEY STAGE 2		
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.		
Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model		
Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.	**Multiplication and division:** Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.	**Fractions:** Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

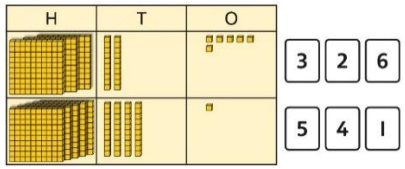
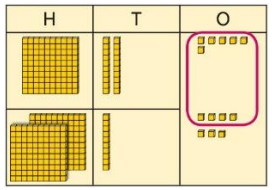
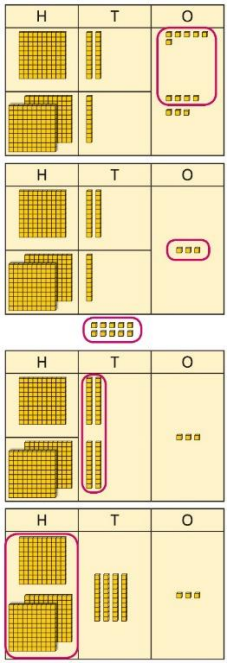
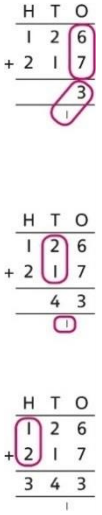
Year 3			
	Concrete	**Pictorial**	**Abstract**
Year 3 Addition			
Understanding 100s	Understand the cardinality of 100, and the link with 10 tens. Use cubes to place into groups of 10 tens. 	Unitise 100 and count in steps of 100. 	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0. 
Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers. 	Use equipment to represent numbers to 1,000.  Use a place value grid to support the structure of numbers to 1,000. Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.	Represent the parts of numbers to 1,000 using a part-whole model.  *215 = 200 + 10 + 5* Recognise numbers to 1,000 represented on a number line, including those between intervals.

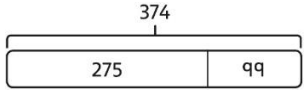
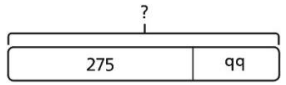
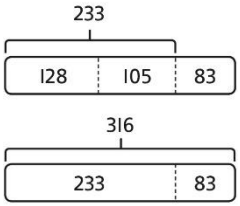
Adding 100s	Use known facts and unitising to add multiples of 100.  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	Use known facts and unitising to add multiples of 100.  *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	Use known facts and unitising to add multiples of 100. Represent the addition on a number line. Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*
3-digit number + 1s, no exchange or bridging	Use number bonds to add the 1s.  *214 + 4 = ?* *Now there are 4 + 4 ones in total.* *4 + 4 = 8* *214 + 4 = 218*	Use number bonds to add the 1s.  *245 + 4* *5 + 4 = 9* *245 + 4 = 249*	Understand the link with counting on. *245 + 4*  Use number bonds to add the 1s and understand that this is more efficient and less prone to error. *245 + 4 = ?* *I will add the 1s.* *5 + 4 = 9* *So, 245 + 4 = 249*

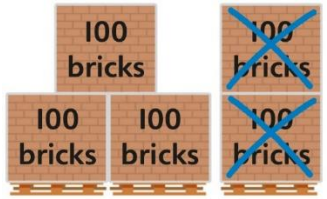
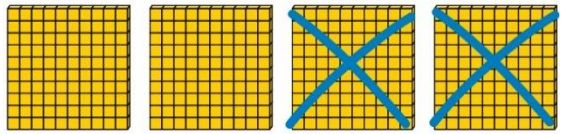
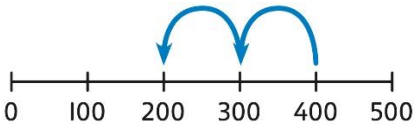
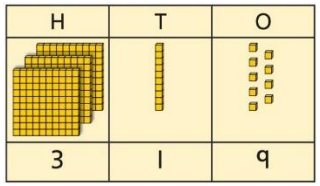
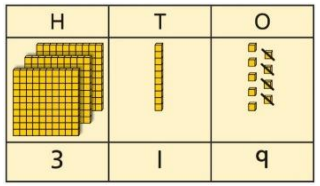
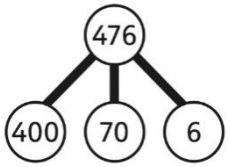
3-digit number + 1s with exchange	Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten. Children should explore this using unitised objects or physical apparatus.	Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.  *135 + 7 = 142*	Understand how to bridge by partitioning to the 1s to make the next 10.  *135 + 7 = ?* *135 + 5 + 2 = 142* Ensure that children understand how to add 1s bridging a 100. *198 + 5 = ?* *198 + 2 + 3 = 203*

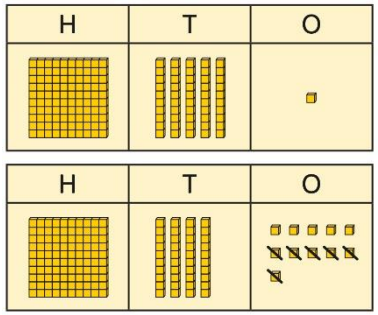
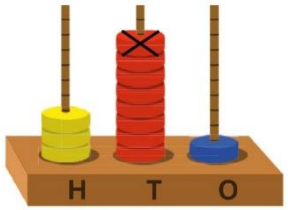
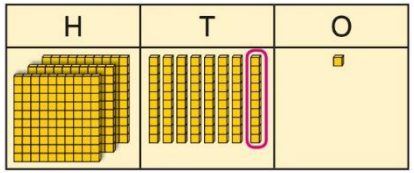
3-digit number + 10s, no exchange	Calculate mentally by forming the number bond for the 10s.  *234 + 50* *There are 3 tens and 5 tens altogether.* *3 + 5 = 8* *In total there are 8 tens.* *234 + 50 = 284*	Calculate mentally by forming the number bond for the 10s. *351 + 30 = ?*  *5 tens + 3 tens = 8 tens* *351 + 30 = 381*	Calculate mentally by forming the number bond for the 10s. *753 + 40* *I know that 5 + 4 = 9* *So, 50 + 40 = 90* *753 + 40 = 793*
3-digit number + 10s, with exchange	Understand the exchange of 10 tens for 1 hundred. 	Add by exchanging 10 tens for 1 hundred. *184 + 20 = ?*  *184 + 20 = 204*	Understand how the addition relates to counting on in 10s across 100.  *184 + 20 = ?* *I can count in 10s … 194 … 204* *184 + 20 = 204* Use number bonds within 20 to support efficient mental calculations. *385 + 50* *There are 8 tens and 5 tens.* *That is 13 tens.* *385 + 50 = 300 + 130 + 5* *385 + 50 = 435*

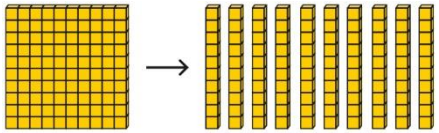
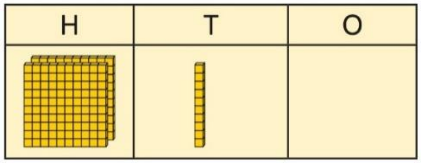
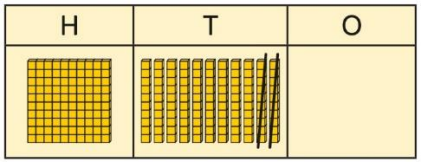
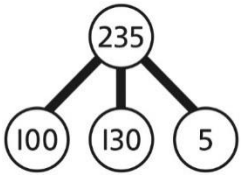
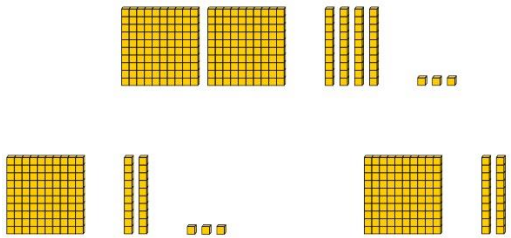
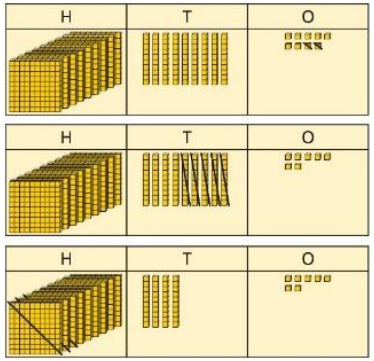
3-digit number + 2-digit number	Use place value equipment to make and combine groups to model addition. 	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.
3-digit number + 2-digit number, exchange required	Use place value equipment to model addition and understand where exchange is required. *Use place value counters to represent 154 + 72.* *Use this to decide if any exchange is required.* *There are 5 tens and 7 tens. That is 12 tens so I will exchange.*	Represent the required exchange on a place value grid using equipment. *275 + 16 = ?*  *275 + 16 = 291* Note: In this example, a mental method may be more efficient. The numbers for the example calculation have been chosen to allow children to visualise the concept and see how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.  *275 + 16 = 291*

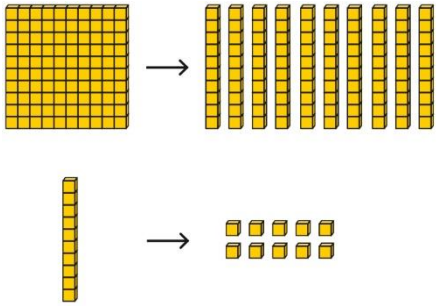
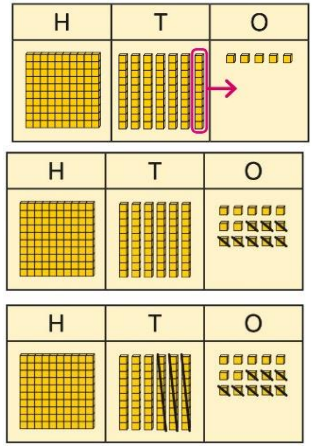
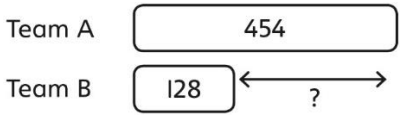
3-digit number + 3-digit number, no exchange	Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. *326 + 541 is represented as:* 	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.
3-digit number + 3-digit number, exchange required	Use place value equipment to enact the exchange required.  *There are 13 ones.* *I will exchange 10 ones for 1 ten.*	Model the stages of column addition using place value equipment on a place value grid. 	Use column addition, ensuring understanding of place value at every stage of the calculation.  *126 + 217 = 343* Note: Children should also study examples where exchange is required in more than one column, for example *185 + 318 = ?*

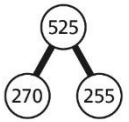
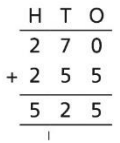
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233 

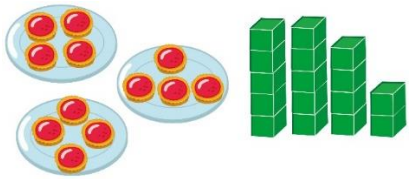
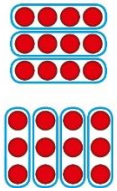
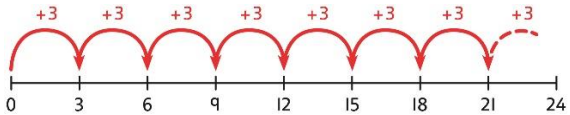
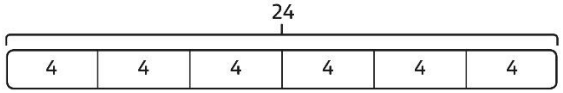
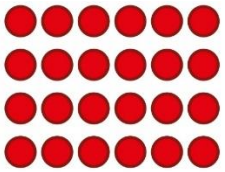
Year 3 Subtraction			
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.  *5 − 2 = 3* *500 − 200 = 300*	Use known facts and unitising to subtract multiples of 100.  *4 − 2 = 2* *400 − 200 = 200*	Understand the link with counting back in 100s.  *400 − 200 = 200* Use known facts and unitising as efficient and accurate methods. *I know that 7 − 4 = 3. Therefore, I know that 700 − 400 = 300.*
3-digit number − 1s, no exchange	Use number bonds to subtract the 1s.  *214 − 3 = ?*  *4 − 3 = 1* *214 − 3 = 211*	Use number bonds to subtract the 1s.  *319 − 4 = ?*  *9 − 4 = 5* *319 − 4 = 315*	Understand the link with counting back using a number line. Use known number bonds to calculate mentally. *476 − 4 = ?*  *6 − 4 = 2* *476 − 4 = 472*

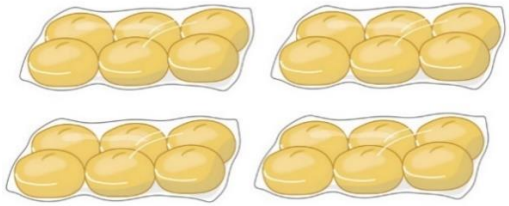
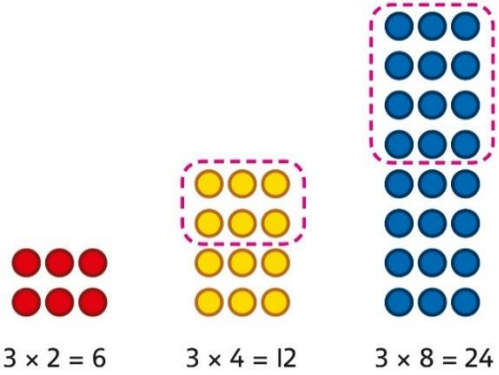
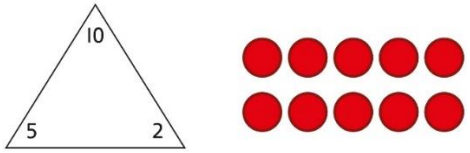
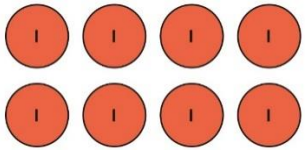
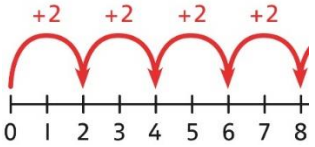
3-digit number − 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. *151 − 6 = ?* 	Calculate mentally by using known bonds. *151 − 6 = ?* *151 − 1 − 5 = 145*
3-digit number − 10s, no exchange	Subtract the 10s using known bonds.  *381 − 10 = ?* *8 tens with 1 removed is 7 tens.* *381 − 10 = 371*	Subtract the 10s using known bonds.  *8 tens − 1 ten = 7 tens* *381 − 10 = 371*	Use known bonds to subtract the 10s mentally. *372 − 50 = ?* *70 − 50 = 20* *So, 372 − 50 = 322*
3-digit number − 10s, exchange or bridging required	Use equipment to understand the exchange of 1 hundred for 10 tens.	Represent the exchange on a place value grid using equipment. *210 − 20 = ?*	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation.

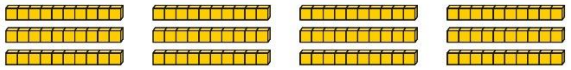
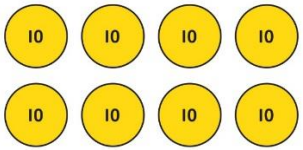
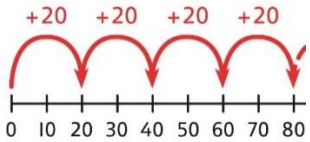
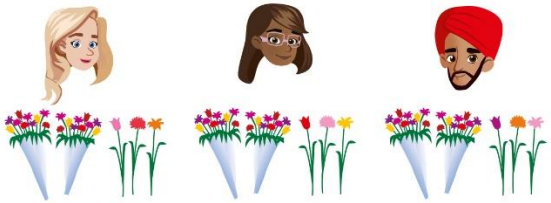
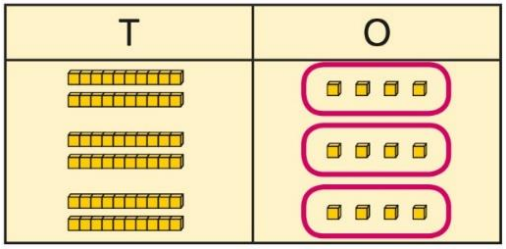
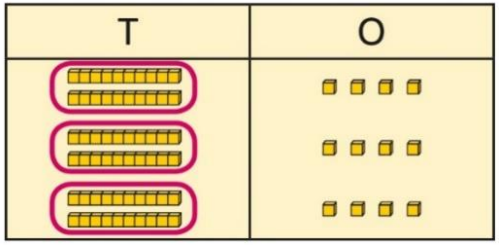
		 *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 − 20 = 190*	*235 − 60 = ?*  *235 = 100 + 130 + 5* *235 − 60 = 100 + 70 + 5* *= 175*
3-digit number − up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away. 	Represent the calculation on a place value grid. 	Use column subtraction to calculate accurately and efficiently. H T O 9 9 9 − 3 5 2 7 H T O 9 9 9 − 3 5 2 4 7 H T O 9 9 9 − 3 5 2 6 4 7
3-digit number − up to 3-digit number, exchange required	Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.	*Model the required exchange on a place value grid.* *175 − 38 = ?*	Use column subtraction to work accurately and efficiently.

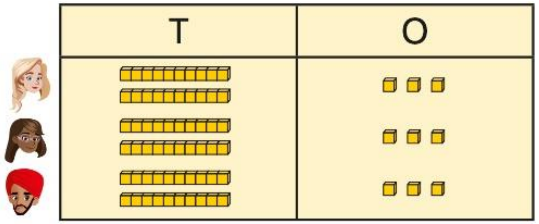
		*I need to subtract 8 ones, so I will exchange a ten for 10 ones.*  	H T O 1 $^{6}\not{7}$ $^{1}5$ − 3 8 1 3 7 175 − 38 = 137 If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly. Children should also understand how to exchange in calculations where there is a zero in the 10s column. 
Representing subtraction problems		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. *I have completed this subtraction.* *525 − 270 = 255* *I will check using addition.*

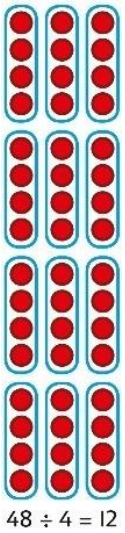
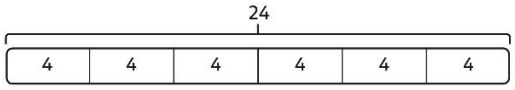
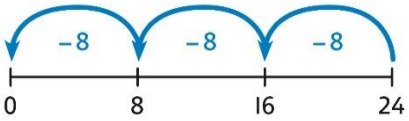
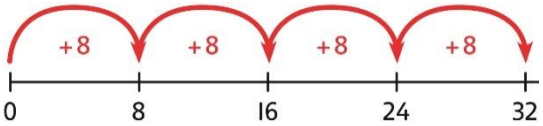
			 

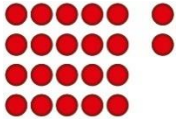
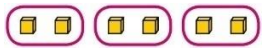
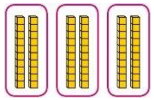
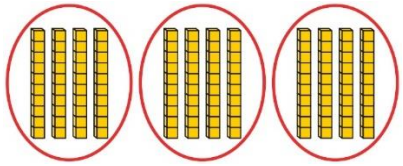
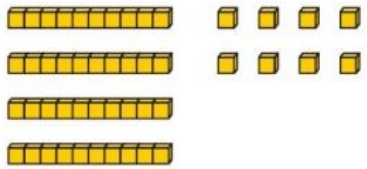
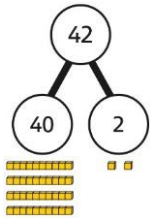
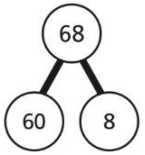
Year 3 Multiplication			
Understanding equal grouping and repeated addition	Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non-examples using objects.  Children recognise that arrays can be used to model commutative multiplications.  *I can see 3 groups of 8.* *I can see 8 groups of 3.*	Children recognise that arrays demonstrate commutativity.  *This is 3 groups of 4.* *This is 4 groups of 3.*	Children understand the link between repeated addition and multiplication.  *8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24* A bar model may represent multiplications as equal groups.  *6 × 4 = 24*
Using commutativity to support understanding of the times-tables	Understand how to use times-tables facts flexibly. 	Understand how times-table facts relate to commutativity. 	Understand how times-table facts relate to commutativity. *I need to work out 4 groups of 7.* *I know that 7 × 4 = 28* *so, I know that*

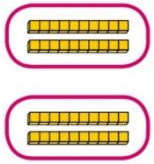
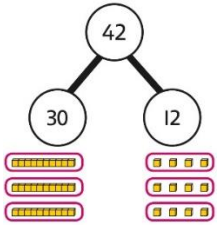
	 *There are 6 groups of 4 pens.* *There are 4 groups of 6 bread rolls.* *I can use 6 × 4 = 24 to work out both totals.*	*6 × 4 = 24* *4 × 6 = 24*	*4 groups of 7 = 28* *and* *7 groups of 4 = 28.*
Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.  *I can use the ×3 table to work out how many keys.* *I can also use the ×3 table to work out how many batteries.*	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.  3 × 2 = 6 3 × 4 = 12 3 × 8 = 24	Children understand the relationship between related multiplication and division facts in known times-tables.  *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. *Make 4 groups of 3 ones.* 	Understand how unitising 10s supports multiplying by multiples of 10. 	Understand how to use known times-tables to multiply multiples of 10. 

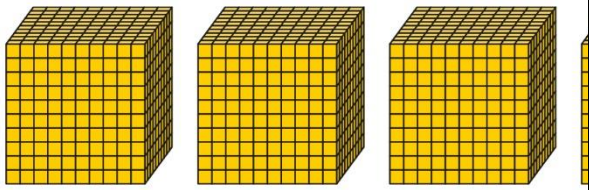
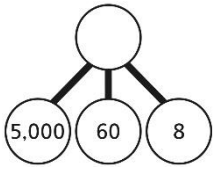
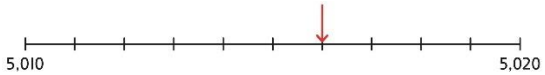
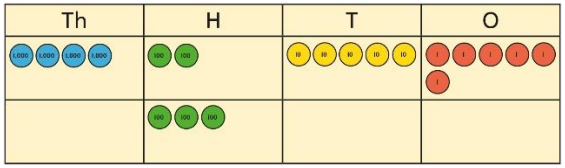
	Make 4 groups of 3 tens.  *What is the same?* *What is different?*	 *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	 *4 × 2 = 8* *4 × 20 = 80*
Multiplying a 2-digit number by a 1-digit number	Understand how to link partitioning a 2-digit number with multiplying. *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*  *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.	Use place value to support how partitioning is linked with multiplying by a 2-digit number. *3 × 24 = ?*  *3 × 4 = 12*  	Use addition to complete multiplications of 2-digit numbers by a 1-digit number. *4 × 13 = ?* *4 × 3 = 12* *4 × 10 = 40* *12 + 40 = 52* *4 × 13 = 52* Use area model as an alternative 16 x 5 = ? × 10 6 5 50 30 50 + 30 = 80

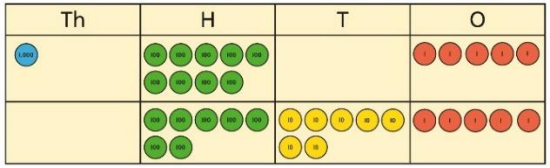
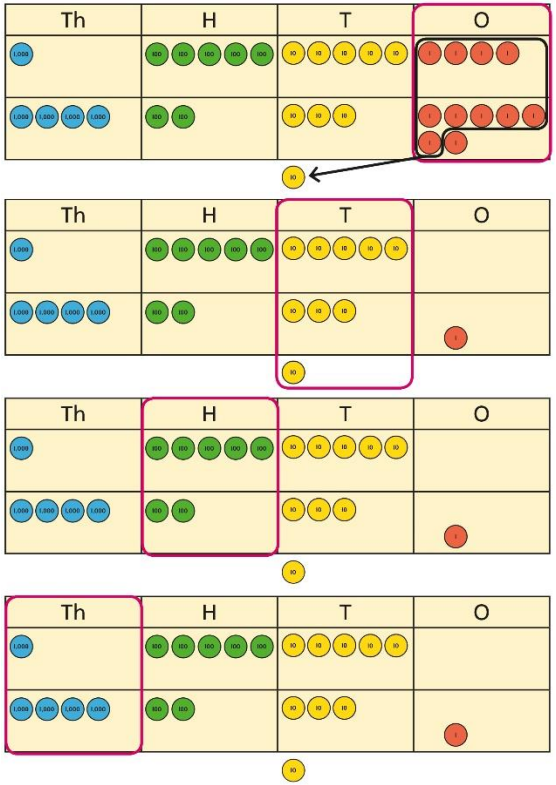
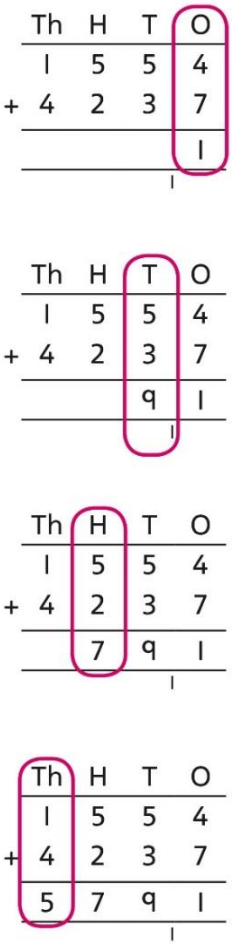
	 *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	*3 × 20 = 60* *60 + 12 = 72* *3 × 24 = 72*	

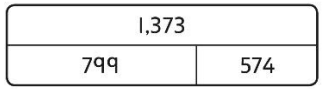
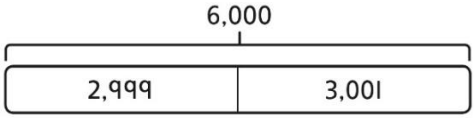
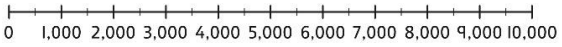
Year 3 Division			
Using times-tables knowledge to divide	Use knowledge of known times-tables to calculate divisions.  *24 divided into groups of 8.* *There are 3 groups of 8.*	Use knowledge of known times-tables to calculate divisions.  48 ÷ 4 = 12 *48 divided into groups of 4.* *There are 12 groups.* *4 × 12 = 48* *48 ÷ 4 = 12*	Use knowledge of known times-tables to calculate divisions. *I need to work out 30 shared between 5.* *I know that 6 × 5 = 30* *so I know that 30 ÷ 5 = 6.* A bar model may represent the relationship between sharing and grouping.  *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition.  *24 ÷ 8 = 3*  *32 ÷ 8 = 4*

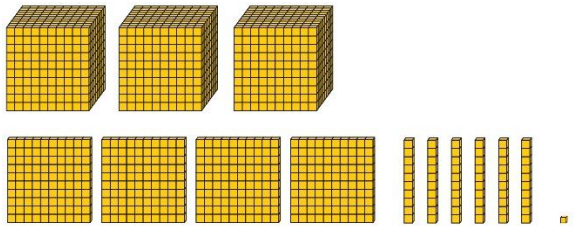
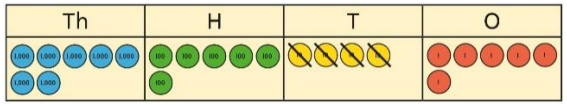
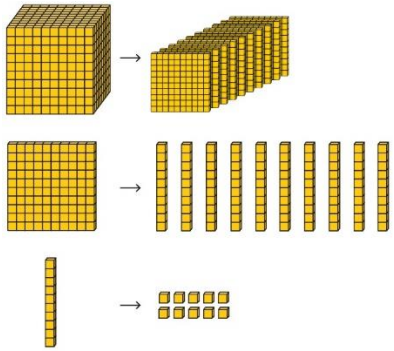
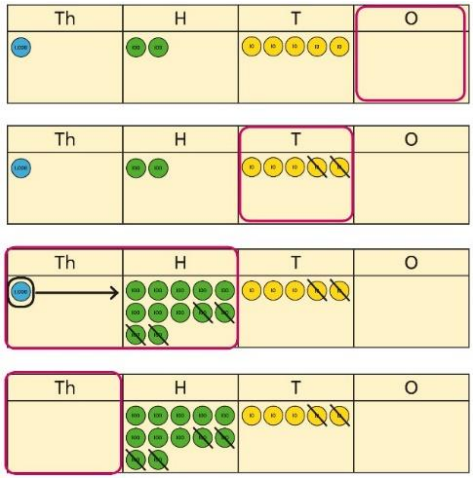
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  *There are 13 sticks in total.* *There are 3 groups of 4, with 1 remainder.*	Use images to explain remainders.  *22 ÷ 5 = 4 remainder 2*	Understand that the remainder is what cannot be shared equally from a set. *22 ÷ 5 = ?* *3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 … this is larger than 22* *So, 22 ÷ 5 = 4 remainder 2*
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. *Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	Divide multiples of 10 by unitising.  *12 tens shared into 3 equal groups.* *4 tens in each group.*	Divide multiples of 10 by a single digit using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment.  *48 ÷ 2 = ?*	Children explore which partitions support particular divisions.  	Children partition a number into 10s and 1s to divide where appropriate.  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34* *68 ÷ 2 = 34*

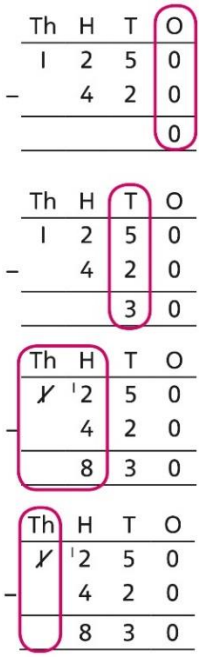
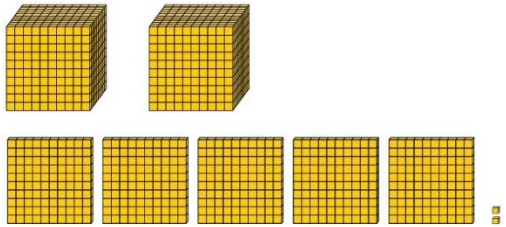
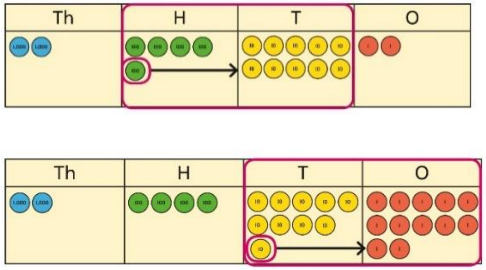
	First divide the 10s.  *Then divide the 1s.* 	*I need to partition 42 differently to divide by 3.*  *42 = 30 + 12* *42 ÷ 3 = 14*	Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12* *30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment. Share it into 2 equal groups*.  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*

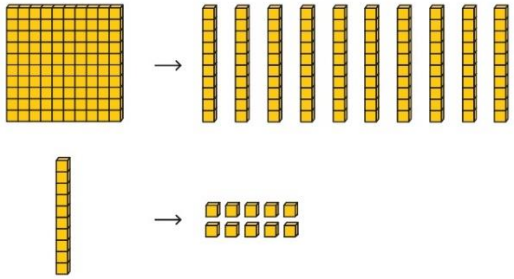
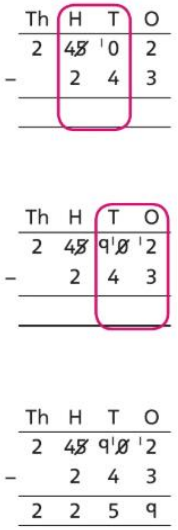
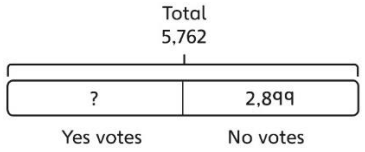
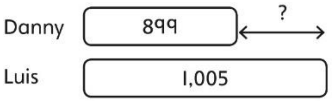
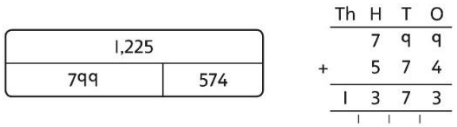
Year 4			
	Concrete	**Pictorial**	**Abstract**
Year 4 Addition			
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.  *4 thousands equal 4,000.* *1 thousand is 10 hundreds.*	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s.  *2,000 + 500 + 40 + 2 = 2,542*	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  *5,000 + 60 + 8 = 5,068* Understand and read 4-digit numbers on a number line. 
Choosing mental methods where appropriate	Use unitising and known facts to support mental calculations. *Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	Use unitising and known facts to support mental calculations.  *I can add the 100s mentally.* *200 + 300 = 500* *So, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5* *200 + 300 = 500* *4,256 + 300 = 4,556*

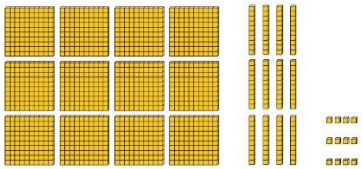
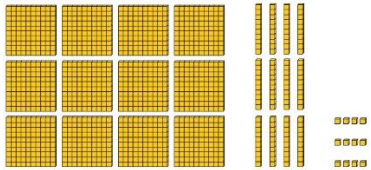
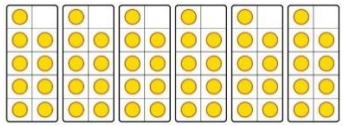
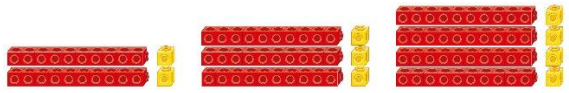
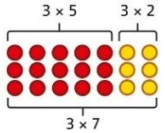
Column addition with exchange	Use place value equipment on a place value grid to organise thinking. Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. *Use equipment.to show 1,905 + 775.*  *Why have only three columns been used for the second row? Why is the Thousands box empty?* *Which columns will total 10 or more?*	Use place value equipment to model required exchanges.  Include examples that exchange in more than one column.	Use a column method to add, including exchanges.  Include examples that exchange in more than one column.

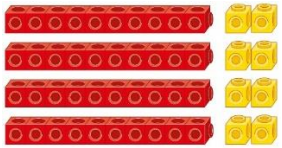
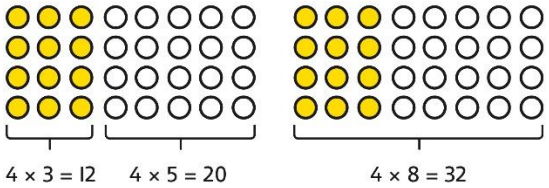
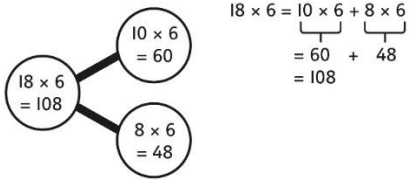
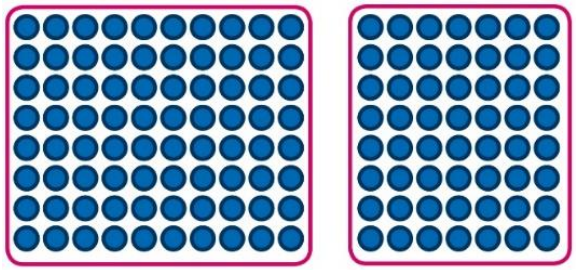
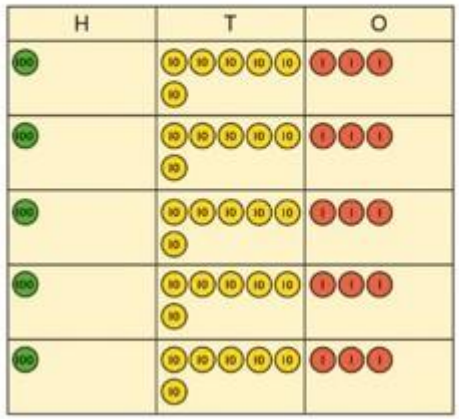
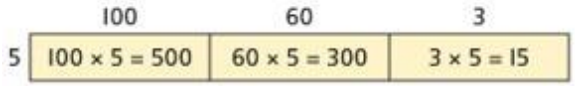
Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.   *I chose to work out 574 + 800, then subtract 1.*  *This is equivalent to 3,000 + 3,000.*	Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.*

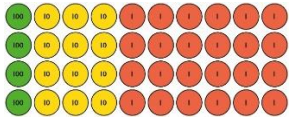
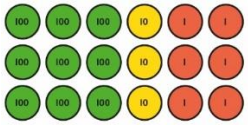
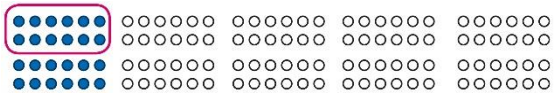
Year 4 Subtraction			
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.  *What number will be left if we take away 300?*	Use place value grids to support mental methods where appropriate.  *7,646 − 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*
Column subtraction with exchange	Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary. 	Represent place value equipment on a place value grid to subtract, including exchanges where needed.  	Use column subtraction, with understanding of the place value of any exchange required.

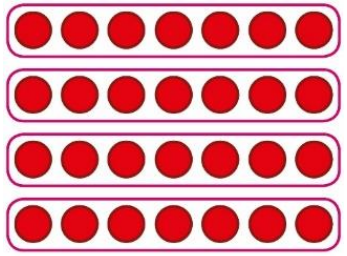
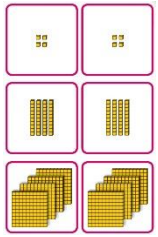
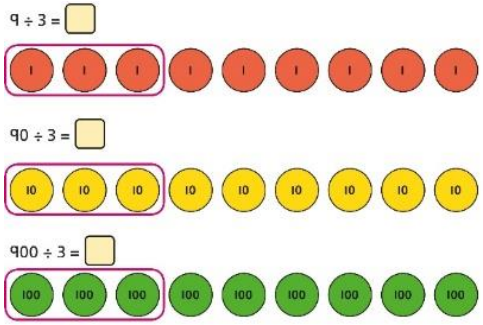
			
Column subtraction with exchange across more than one column	Understand why two exchanges may be necessary. *2,502 − 243 = ?*  *I need to exchange a 10 for some 1s, but there are not any 10s here.*	Make exchanges across more than one column where there is a zero as a place holder. *2,502 − 243 = ?* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 − 243 = ?*

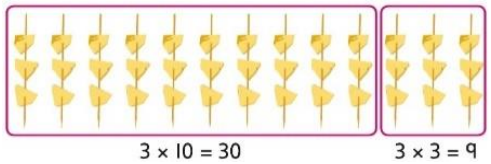
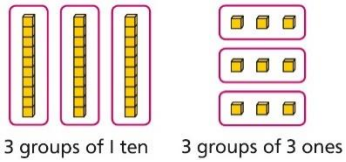
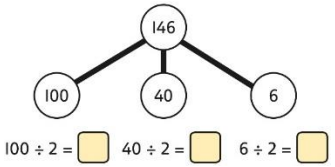
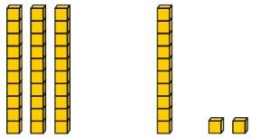
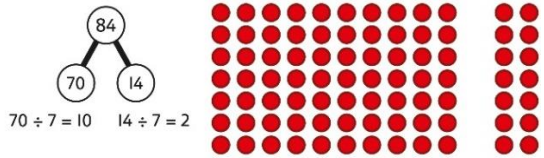
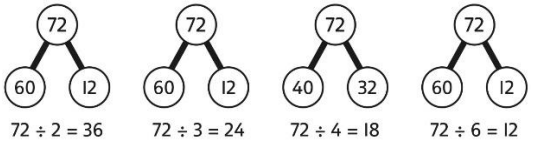
			
Representing subtractions and checking strategies		Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 − 2,899.* Bar models can also represent 'find the difference' as a subtraction problem. 	Use inverse operations to check subtractions. *I calculated 1,225 − 799 = 574.* *I will check by adding the parts.*  *The parts do not add to make 1,225.* *I must have made a mistake.*

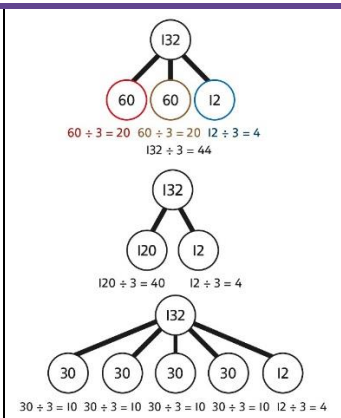
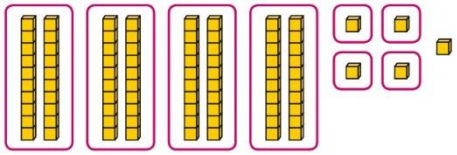
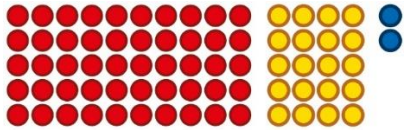
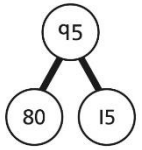
Year 4 Multiplication			
Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	Use known facts and understanding of place value and commutativity to multiply mentally. *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.  *5 × 1 = 5* *5 × 0 = 0*	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table *I know that 7 × 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 − 6*

Understanding and using partitioning in multiplication	Make multiplications by partitioning. *4 × 12 is 4 groups of 10 and 4 groups of 2.*  *4 × 12 = 40 + 8*	Understand how multiplication and partitioning are related through addition.  *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	Use partitioning to multiply 2-digit numbers by a single digit. *18 × 6 = ?*  *18 × 6 = 10 × 6 + 8 × 6* *= 60 + 48* *= 108*
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Explore how to use partitioning to multiply efficiently. *8 × 17 = ?*  80 + 56 = 136	Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.  	Use an area model and then add the parts.  500 + 300 + 15 = 815 8 x 256 = ? **200 50 6** **8** **1600 400 48** **= 2048**

Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications. *Make 4 × 136 using equipment.*  *I can work out how many 1s, 10s and 100s.* *There are 4 × 6 ones… 24 ones* *There are 4 × 3 tens … 12 tens* *There are 4 × 1 hundreds … 4 hundreds* *24 + 120 + 400 = 544*	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  3 1 2 × 3 9 3 6	Use the formal column method for up to 3-digit numbers multiplied by a single digit. 3 1 2 × 3 9 3 6 Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. 2 3 × 5 1 5 1 0 0 1 1 5 2 3 × 5 1 1 5 1
Multiplying more than two numbers	Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* 5 × 2 × 3 = 30 10 × 3 = 30	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* *60 × 2 = 120*	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5* 12 × 2 × 5 = 12 × 10 = 120 So, 24 × 5 = 120

Year 4 Division			
Understanding the relationship between multiplication and division, including times-tables	Use objects to explore families of multiplication and division facts.  *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*	Represent divisions using an array.  28 ÷ 7 = 4	Understand families of related multiplication and division facts. *I know that 5 × 7 = 35* *so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.  *8 ones divided into 2 equal groups* *4 ones in each group* *8 tens divided into 2 equal groups* *4 tens in each group* *8 hundreds divided into 2 equal groups* *4 hundreds in each group*	Represent divisions using place value equipment.  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*

Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* *142 ÷ 2 = 73*
Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.* 	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.*  84 ÷ 7 = 12	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division.

			
Understanding remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	Represent the remainder as the part that cannot be shared equally.  *72 ÷ 5 = 14 remainder 2*	Understand how partitioning can reveal remainders of divisions.  *80 ÷ 4 = 20* *12 ÷ 4 = 3* *95 ÷ 4 = 23 remainder 3*